

ON THE DYNAMICS OF FOLIATIONS IN \mathbb{P}^n TANGENT TO LEVI-FLAT HYPERSURFACES

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ABSTRACT. Let \mathcal{F} be a codimension one holomorphic foliation in \mathbb{P}^n , $n \geq 2$, leaving invariant a real analytic Levi-flat hypersurface M with regular part M^* . Then every leaf of \mathcal{F} outside $\overline{M^*}$ accumulates in $\overline{M^*}$.

1. INTRODUCTION

Let $M \subset U \subset \mathbb{C}^n$ be a real analytic variety of real codimension one, where U is an open set. Let M^* denote its *regular* part, that is, the smooth part of M of highest dimension — near each point $x \in M^*$, the variety M is a manifold of real codimension one. For each $x \in M^*$, there is a unique complex hyperplane \mathcal{L}_x contained in the tangent space $T_x M^*$. This defines a real analytic distribution $x \mapsto \mathcal{L}_x$ of complex hyperplanes in TM^* , known as the *Levi distribution*. When this distribution is integrable in the sense of Frobenius, we say that M is a *Levi-flat* hypersurface. The resulting foliation, denoted by \mathcal{L} , is known as the *Levi foliation*. This concept goes back to E. Cartan, who proved that there are local holomorphic coordinates (z_1, \dots, z_n) around $x \in M^*$ such that $M^* = \{\operatorname{Im}(z_n) = 0\}$ ([8], Théorème IV). As a consequence, the leaves of the Levi foliation \mathcal{L} have local equations $z_n = c$, for $c \in \mathbb{R}$. From the global viewpoint, they are complex manifolds of codimension one immersed in U . Cartan's local trivialization provides an intrinsic way to extend the Levi foliation to a non-singular holomorphic foliation in a neighborhood of M^* . Locally, we extend \mathcal{L} to a neighborhood of $x \in M^*$ as the foliation having, in the coordinates (z_1, \dots, z_n) , horizontal leaves $z_n = c$, for $c \in \mathbb{C}$. Since M^* has real codimension 1, this is the unique possible local extension of \mathcal{L} , so that these local extensions glue together yielding a foliation defined in

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whole neighborhood of M^* . Nevertheless, it is not true in general that \mathcal{L} extends to a holomorphic foliation in a neighborhood of $\overline{M^*}$, even if singularities are admitted. There are examples of Levi-flat hypersurfaces whose Levi foliations extend to k -webs in the ambient space (see [2] and [12]). When there is a holomorphic foliation \mathcal{F} in U which restricted to M^* is the Levi foliation \mathcal{L} we say either that M is *invariant* by \mathcal{F} or that \mathcal{F} leaves M *invariant*.

Local problems about singular Levi-flat hypersurfaces have been studied by many authors — see for instance [1], [11], [13], [16] and the references within. Germs of codimension one foliations at $(\mathbb{C}^n, 0)$ leaving invariant real-analytic Levi-flat hypersurfaces are well understood: they are given by the levels of meromorphic functions — possibly holomorphic — according to a theorem by D. Cerveau and A. Lins Neto (see [9] and also [3]). In this note, our object is a globally defined singular holomorphic foliation \mathcal{F} in \mathbb{P}^n , $n \geq 2$, having an invariant real analytic Levi-flat hypersurface M . Our main result asserts that Levi-flat hypersurfaces in \mathbb{P}^n are attractors for the ambient foliation, in the sense that the leaves of \mathcal{F} accumulates in $\overline{M^*}$. This is the content of:

Theorem I. *Let \mathcal{F} be a complex foliation of codimension one in \mathbb{P}^n , with $n \geq 2$, leaving invariant a real analytic Levi-flat hypersurface M . Then every leaf of \mathcal{F} accumulates in $\overline{M^*}$.*

We give a sketch of the proof of Theorem I, starting with the result in dimension two. First of all, by [15], the components of $\mathbb{P}^2 \setminus \overline{M^*}$ are Stein Varieties, which, by a theorem of Hörmander, are properly embedded in the affine space \mathbb{C}^5 . If there existed a leaf L of \mathcal{F} whose closure does not intersect $\overline{M^*}$, then \overline{L} should contain a singular point of \mathcal{F} , otherwise \overline{L} would yield a minimal set contained in a Stein variety, which is not allowed. A leaf that accumulates in a singular point either contains a separatrix in its closure or is contained in a nodal separator (see [6]). This allows us to use the standard Maximum Modulus Principle or its version for nodal separators contained in Proposition 2 to reach a contradiction with the compactness of \overline{L} . Finally, the n -dimensional result is obtained from the two-dimensional version by restricting the foliation to a generic two-dimensional plane.

2. PRELIMINARIES

A holomorphic foliation of codimension one and degree d in \mathbb{P}^n is induced, in homogeneous coordinates $(Z_0 : Z_1 : \dots : Z_n)$, by a 1-form ω whose coefficients are homogeneous polynomials of degree $d+1$ satisfying the following conditions:

- (i) $\omega \wedge d\omega = 0$ (integrability);
- (ii) $i_{\mathbf{r}}\omega = \sum_{i=0}^n X_i A_i(X) = 0$, where $\mathbf{r} = X_0 \partial/\partial X_0 + \cdots X_n \partial/\partial X_n$ is the radial vector field (Euler's condition);
- (iii) $\text{codim Sing}(\mathcal{F}) \geq 2$,

where $\text{Sing}(\mathcal{F}) = \{A_0 = A_1 = \cdots = A_n = 0\}$ is the *singular set* of \mathcal{F} . This means that ω defines, outside $\text{Sing}(\mathcal{F})$, a regular foliation of codimension one in \mathbb{C}^{n+1} whose leaves are tangent to the distribution of tangent spaces given by ω . Euler's condition assures that this foliation goes down to \mathbb{P}^n .

Let M be an irreducible singular real-analytic Levi-flat hypersurface in the complex projective space \mathbb{P}^n . Let M^* be the regular part of M , that is, the set of points near which M is a nonsingular real-analytic hypersurface of real codimension one. We denote by $\text{Sing}(M)$ the singular points of M , points near which M is not a real-analytic submanifold of any dimension. Note that, in general, $M^* \cup \text{Sing}(M) \subsetneq M$.

In our approach to global Levi-flat hypersurfaces in \mathbb{P}^n , an important tool is the use of geometric and analytic properties of Stein manifolds. Actually, we have the following result (see [15]), which will play an essential role in the proof of Theorem I:

Theorem 1. *Let $M \subset \mathbb{P}^n$ be a real-analytic Levi-flat hypersurface. Suppose that for every $p \in \overline{M^*}$ there exists a neighborhood U_p and a meromorphic function F_p defined in U_p which is constant along the leaves of M^* . Then all the connected components of $\mathbb{P}^n \setminus \overline{M^*}$ are Stein.*

This result follows from a theorem of Takeuchi which asserts that an open set $U \subset \mathbb{P}^n$ which is pseudoconvex is Stein (see [20]). The hypothesis on the existence of local meromorphic first integrals in the ambient assures that, at every point $p \in \overline{M^*}$, there exists a germ of complex hypervariety contained in $\overline{M^*}$. This implies pseudoconvexity for all connected components of $\mathbb{P}^n \setminus \overline{M^*}$. Notice that, following Cerveau-Lins Neto's Theorem, such a condition is naturally fulfilled when M is tangent to a global foliation in \mathbb{P}^n . Conversely, the existence of local meromorphic first integrals allows a natural extension of the Levi foliation to a neighborhood of $\overline{M^*}$ and a subsequent extension to the whole \mathbb{P}^n , since $\mathbb{P}^n \setminus \overline{M^*}$ has components which are Stein (see [17]).

A strong motivation to the theory of Levi-flat hypersurfaces is the study of minimal sets for foliations (see [4], [10]). If \mathcal{F} is a singular foliation of dimension r in a complex manifold X of dimension $n > r$, then a compact non-empty subset $\mathcal{M} \subset X$ is said to be a *minimal set* for \mathcal{F} if the following properties are satisfied:

- (i) \mathcal{M} is invariant by \mathcal{F} ;
- (ii) $\mathcal{M} \cap \text{Sing}(\mathcal{F}) = \emptyset$;
- (iii) \mathcal{M} is minimal with respect to these properties.

Notice that if L is a leaf of \mathcal{F} such that \overline{L} is compact and $\overline{L} \cap \text{Sing}(\mathcal{F}) = \emptyset$, then \overline{L} contains a minimal set for \mathcal{F} . If X is a Stein manifold, then \mathcal{F} contains no minimal sets. Indeed, a Stein manifold admits a C^∞ strictly plurisubharmonic function ϕ . If a minimal set \mathcal{M} existed, then the restriction of ϕ to \mathcal{M} would assume a maximum value at a point $p \in \mathcal{M}$. If L_p is the leaf of \mathcal{F} containing p , the maximum principle for the plurisubharmonic function $\phi|_{L_p}$ would force ϕ to be constant over L_p , contradicting its strict plurisubharmonicity. This fact has the following consequence:

Proposition 1. *Let \mathcal{F} be a complex foliation in \mathbb{P}^n leaving invariant a real analytic Levi-flat hypersurface M . Let L be a leaf of \mathcal{F} . Then either $\overline{L} \cap \overline{M}^* \neq \emptyset$ or $\overline{L} \cap \text{Sing}(\mathcal{F}) \neq \emptyset$.*

Proof. If neither of the alternatives were true, then \overline{L} would be a minimal set contained in a Stein variety. This is not allowed, as we commented above. \square

For codimension one foliations, the existence of a non-singular Levi-flat hypersurfaces would imply the existence of a minimal set. In \mathbb{P}^n , for $n \geq 3$, there are neither non-singular real-analytic Levi-flat hypersurfaces, nor minimal sets, as proved in [17]. In dimension two, however, the existence of both real-analytic Levi-flats and minimal sets are so far open problems.

3. NODAL SEPARATORS

A singular point p for a local foliation \mathcal{F} at (\mathbb{C}^2, p) is *simple* if, given a vector field \mathbf{v} that induces \mathcal{F} around p , the linear part $D\mathbf{v}(p)$ of \mathbf{v} at p has eigenvalues λ_1 and λ_2 satisfying one of the two possibilities:

- (i) $\lambda_1 \neq 0$ and $\lambda_2 = 0$;
- (ii) $\lambda_1 \lambda_2 \neq 0$ and $\lambda_1/\lambda_2 \notin \mathbb{Q}^+$.

Model number (i) above is called *saddle node*, whereas a singularity satisfying (ii) is said to be *non-degenerate*. Seidenberg's Desingularization Theorem asserts that there is a finite sequence of punctual blow-ups $\pi : (M, E) \rightarrow (\mathbb{C}^2, p)$, where $E = \pi^{-1}(0)$ is a normal crossings divisor of projective lines and M is a germ of complex surface around E , for which $\pi^*\mathcal{F}$, the strict transform of \mathcal{F} , is a foliation in M whose singularities on E are all simple. We say that a germ of complex foliation

\mathcal{F} with isolated singularity at $(\mathbb{C}^2, 0)$ is a *generalized curve* if there are no saddle-nodes in its desingularization (see [7]).

If $p \in \text{Sing}(\mathcal{F})$ is a saddle-node, then there exists a smooth invariant curve S invariant by \mathcal{F} containing p , corresponding to the non-zero eigenvalue, the so-called strong separatrix — to the zero eigenvalue is associated a formal, possibly non-convergent, invariant curve named *weak separatrix*. In a neighborhood of $S \setminus \{p\}$, all leaves accumulate in S , so that they are not closed in $U \setminus \{p\}$ for some neighborhood U of p . This is incompatible, for instance, with the existence of a local holomorphic first integral for \mathcal{F} , since, in this case, all leaves near p would be closed (see, for instance [19]). Evidently, a germ of foliation \mathcal{F} at (\mathbb{C}^2, p) which admits a meromorphic first integral is a generalized curve, since its desingularization produces simple singularities admitting holomorphic first integrals. It follows from Ceveau-Lins Neto's Theorem that a germ of foliation \mathcal{F} at (\mathbb{C}^2, p) leaving invariant a germ of Levi-flat hypersurface is a generalized curve.

Now, suppose that \mathcal{F} is a local foliation with a simple singularity at $p \in \mathbb{C}^2$ having non-zero eigenvalues λ_1, λ_2 such that $\lambda = \lambda_2/\lambda_1 \in \mathbb{R}^+ \setminus \mathbb{Q}^+$. Such a singularity is called *node*. The condition on the eigenvalues puts this singularity in the Poincaré domain, so that \mathcal{F} may be linearized by holomorphic coordinates (x, y) , being defined by the linear 1-form

$$\omega = -\lambda y dx + x dy.$$

The multivalued function $f_\lambda = x^{-\lambda}y$ is a first integral for \mathcal{F} . Outside the separatrices $x = 0$ and $y = 0$, the function $|f_\lambda|$ is real analytic, so that $S_c : |f_\lambda| = c$, for a fixed $c \in \mathbb{R}_+$, is hypersurface, real analytic outside the separatrices, which is foliated by the leaves of \mathcal{F} , all of them dense in S_c . This set has the property that its complement in a small neighborhood of p minus the local separatrices is not connected. It thus works as a barrier, preventing local leaves in one component to pass to the other. The set S_c is called *nodal separator*, following the terminology of D. Marín and J.-F. Mattei (see [18]). The image of a nodal separator by a desingularization map is also called a nodal separator, so that this concept extends to non-simple singularities.

Sets with separation properties such as nodal separators do not exist for non-nodal simple non-degenerate singularities. Actually, when \mathcal{F} is simple non-degenerate at $p \in \mathbb{C}^2$, then \mathcal{F} has two separatrices. If, besides, $p \in \mathbb{C}^2$ is non-nodal, then the union of one of the separatrices with the *saturation* of a small complex disc Σ transversal to the other one — that is, the union of all leaves of \mathcal{F} intersecting Σ — is a neighborhood of p . Intuitively, leaves that are sufficiently near one of the

separatrices approach the other one. Invariant objects with separation properties such as nodal separators also fail to exist when $p \in \mathbb{C}^2$ is a saddle node.

The main result in [6] asserts that if \mathcal{F} is a germ of holomorphic foliation at (\mathbb{C}^2, p) with isolated singularity and $\mathcal{I} \neq \{p\}$ is a closed invariant subset such that $p \in \mathcal{I}$, then \mathcal{I} contains either a separatrix or a nodal separator at p . In other words, if L is a local leaf of \mathcal{F} such that $p \in \overline{L}$, take $\mathcal{I} = \overline{L}$ in order to conclude that \overline{L} contains either a separatrix or a nodal separator at $p \in \mathbb{C}^2$.

Both separatrices and nodal separators are object of maximum modulus properties which will be useful to our purposes. In the case of a separatrix, we have the following: if a germ of analytic function $f \in \mathcal{O}_0$ and a germ of analytic curve S at $(\mathbb{C}^2, 0)$ are such that $|f|_S$ has a local maximum at some point $p \in S$ then f is constant over S . This follows from the standard Maximum Modulus Principle, after possibly desingularizing S . Next, we propose a version of the Maximum Modulus Principle for the universe of nodal separators.

Proposition 2. *Let S be a nodal separator for a local foliation \mathcal{F} in $(\mathbb{C}^2, 0)$. Let $f \in \mathcal{O}_0$ be a holomorphic germ of function such that $|f|_S$ has a maximum at $0 \in \mathbb{C}^2$. Then f is constant.*

Proof. By taking a desingularization, we may assume that $0 \in \mathbb{C}^2$ is a simple singularity, with local coordinates (x, y) such that S has the equation $|y| = |x|^\lambda$. We only need to prove that $|f|$ is constant over S . Actually, if this is so, then f will be constant on each leaf of \mathcal{F} contained in S . Each of these leaves accumulates in $0 \in \mathbb{C}^2$, so that $f \equiv f(0)$ on S . This shows that the analytic set $f = f(0)$ contains the non-analytic set S , which implies that $f \equiv f(0)$.

Let U be a neighborhood of $0 \in \mathbb{C}^2$, with $S \cap U$ connected, such that $|f(0)| \geq |f(p)|$ for all $p \in S \cap U$. Notice that if there existed $p \in S \cap U$ with $p \neq 0$ such that $|f(p)| = |f(0)|$, then f would be constant on the leaf L_p of \mathcal{F} containing p . This leaf accumulates in 0 , so that $f \equiv f(0)$ on L_p . Again, the analytic set $f = f(0)$ would contain the non-analytic set $L_p \cup \{0\}$, which would imply $f \equiv f(0)$. Thus, we can suppose that $|f(0)| > |f(p)|$ for all $p \in S \cap U$. By the continuity of f , it is possible to obtain a neighborhood V of $S \setminus \{0\}$, $V \subset U$, such that $|f(0)| > |f(p)|$ for all $p \in V$.

Let us fix $\epsilon > 0$ and a closed annulus $A = \{x \in \mathbb{C}; \rho_1 \leq |x| \leq \rho_2\}$, for some $0 < \rho_1 < \rho_2$, such that (x, y) in V whenever $x \in A$ and $||y| - |x|^\lambda| < \epsilon$. Thus, if $p/q \in \mathbb{Q}_+$ is sufficiently near λ , then all the determinations of $(x, y^{p/q})$ will lie in V for all $x \in A$. Let $S_{p/q}$ be the analytic curve of equation $y^q = x^q$. We have that $0 \in S_{p/q}$ and, over

the annulus A , $S_{p/q} \subset V$. Therefore, the maximum of f over $S_{p/q}$, for $|x| \leq \rho_2$, is reached at some point $(x_0, y_0) \in S_{p/q}$ with $|x_0| < \rho_1$. Now, the Maximum Modulus Principle applied to the analytic curve $S_{p/q}$ gives that $f \equiv f(0)$ over $S_{p/q}$. Finally, curves such as $S_{p/q}$ are dense in the nodal separator, which allows us to conclude that f is constant over S . \square

4. PROOF OF THEOREM I

We are now ready to proof Theorem I:

Theorem I. *Let \mathcal{F} be a complex foliation of codimension one in \mathbb{P}^n , with $n \geq 2$, leaving invariant a real analytic Levi-flat hypersurface M . Then every leaf of \mathcal{F} accumulates in \overline{M}^* .*

Proof. We first suppose that \mathcal{F} is a foliation in \mathbb{P}^2 . Let us suppose, by contradiction, that there exists a leaf L such that $\overline{L} \cap \overline{M}^* = \emptyset$. Then \overline{L} is a compact set contained in a connected component W of $\mathbb{P}^2 \setminus \overline{M}^*$, which is a two-dimensional Stein variety. Notice that, by Proposition 1, necessarily \overline{L} intersects $\text{Sing}(\mathcal{F})$. A theorem of Hörmander assures the existence of a proper holomorphic embedding $\phi : W \rightarrow \mathbb{C}^5$ (see [14]). The restriction of \mathcal{F} to W is carried by ϕ to a foliation in the closed surface $\phi(W) \subset \mathbb{C}^5$. For simplicity, we use the same notation for \mathcal{F} and its leaves in $W \subset \mathbb{P}^2$ and for their images in $\phi(W) \subset \mathbb{C}^5$. Take a non-zero $\mathbf{v} \in \mathbb{C}^5$ and define

$$\begin{aligned} f = f_{\mathbf{v}} : \mathbb{C}^5 &\longrightarrow \mathbb{C} \\ z &\longmapsto \langle z, \mathbf{v} \rangle, \end{aligned}$$

where $\langle z, \mathbf{v} \rangle = z_1 v_1 + \dots + z_5 v_5$, $z = (z_1, \dots, z_5)$ and $\mathbf{v} = (v_1, \dots, v_5)$. Since $\overline{L} \subset \mathbb{C}^5$ is compact, there exists $p \in \overline{L}$ where $|f|_{\overline{L}}$ reaches its maximum value.

Assertion: There exists a leaf L_1 of \mathcal{F} with $L_1 \subset \overline{L}$, such that $f|_{L_1}$ is constant.

We consider two cases:

1st case: p is a regular point for \mathcal{F} . Take L_1 the leaf of \mathcal{F} containing p . Since $L_1 \subset \overline{L}$, the function $|f|_{L_1}$ has a maximum value at p . Thus, $f|_{L_1}$ is constant by the Maximum Value Principle.

2nd case: $p \in \text{Sing}(\mathcal{F})$. In this case, by [6], \overline{L} contains either a separatrix or a nodal separator for \mathcal{F} at p . Take L_1 to be the separatrix, in the first case, or one of the leaves contained in the nodal separator, in the second. In both cases, $f|_{L_1}$ will be constant by the Maximum Value Principle (Proposition 2, for the nodal separator)

The leaf L_1 found above is such that f is constant over $\overline{L_1}$. Thus, $\overline{L_1} \subset f^{-1}(f(p))$, which is a complex hyperplane in \mathbb{C}^5 , that is, an affine space isomorphic to \mathbb{C}^4 . Repeating this procedure three more times, we will eventually find a leaf $L_0 \subset \overline{L}$ such that $\overline{L_0} \subset \mathbb{C}$. This gives a contradiction with the compactness of $\overline{L_0}$.

Now, suppose that \mathcal{F} is a foliation in \mathbb{P}^n , with $n \geq 3$, leaving invariant a real-analytic Levi-flat hypersurface M . Let L be a leaf outside $\overline{M^*}$. By [5], we can choose a linear embedding $i : \mathbb{P}^2 \hookrightarrow \mathbb{P}^n$ transversal to \mathcal{F} and to L — that is, $i^*\mathcal{F}$ is a foliation with isolated singularities having $i^{-1}(L)$ as a leaf. Notice that $i^{-1}(M)$ is a real-analytic Levi-flat hypersurface whose Levi foliation is $i^*\mathcal{L}$, where \mathcal{L} is the Levi foliation on M . Now, we apply the two-dimensional case in order to conclude that $i^{-1}(L)$ intersects the closure of the regular part of $i^{-1}(M)$. This give straight that $\overline{L} \cap \overline{M^*} \neq \emptyset$. \square

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